

LESSON SLIDES

Integers

Maths · Number line, the four operations and key properties

By the end of this lesson you can

- 🎯 Show and compare integers on a number line
- 🎯 Add and subtract integers without sign errors
- 🎯 Multiply and divide integers using sign rules
- 🎯 Use the properties of integer operations
- 🎯 Solve problems on lifts, temperature and money

What are integers?

- Integers: ..., -3 , -2 , -1 , 0 , 1 , 2 , 3 , ...
- Positive integers are greater than 0
- Negative integers are less than 0
- Zero is neither positive nor negative
- 2.5 and $\frac{1}{2}$ are not integers

Integers in daily life

- Lift: ground floor is 0, first basement is -1
- Temperature: 5°C below zero is -5°C
- Money: a debt of USD 200 is -200
- Height: 50 m below sea level is -50 m

Comparing integers on a number line

- Values increase as you move right
- $-2 > -5$, because -2 lies to the right
- Every negative integer is less than 0
- Every positive integer is greater than every negative

Adding integers: think of a lift

- Add a positive: move up, to the right
- Add a negative: move down, to the left
- Floor 2 + $(-3) = -1$, one level below ground
- $(-4) + (-3) = -7$ and $6 + (-8) = -2$

Two rules for adding integers

Same signs

- Add the sizes
- Keep the common sign
- $(-4) + (-3) = -7$
- $5 + 2 = 7$

Different signs

- Subtract the smaller size from the larger
- Keep the sign of the larger size
- $(-8) + 5 = -3$
- $9 + (-4) = 5$

Subtracting integers: add the opposite

- The additive inverse of 7 is -7 , and of -7 is 7
- $a - b$ means $a + (\text{inverse of } b)$
- $5 - (-3) = 5 + 3 = 8$
- $(-2) - 4 = (-2) + (-4) = -6$
- From $3\text{ }^{\circ}\text{C}$ to $-4\text{ }^{\circ}\text{C}$: $(-4) - 3 = -7\text{ }^{\circ}\text{C}$ change

Multiplying integers: spot the pattern

- $3 \times (-4) = -12$ and $2 \times (-4) = -8$
- $1 \times (-4) = -4$ and $0 \times (-4) = 0$
- Each step adds 4, so $(-1) \times (-4) = 4$
- A negative multiplier reverses the direction

Sign rules for $\times$ and $\div$

Same signs: positive

- $(+) \times (+) = (+)$
- $(-) \times (-) = (+)$
- $(-6) \times (-5) = 30$
- $(-100) \div (-4) = 25$

Different signs: negative

- $(+) \times (-) = (-)$
- $(-) \times (+) = (-)$
- $7 \times (-3) = -21$
- $(-100) \div 25 = -4$

Dividing integers

- Division undoes multiplication
- $(-100) \div 25$ asks: $25 \times ? = -100$, so -4
- $(-48) \div 6 = -8$ and $(-48) \div (-6) = 8$
- Division by zero is not defined

Early rules for negative numbers

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CE: Brahmagupta wrote rules for multiplying and dividing negatives

- He called positive numbers fortunes and negative numbers debts
- Product of two debts is a fortune: $(-) \times (-) = (+)$

Properties of integer operations

- Closure: $+$, $-$ and $\times$ of integers give an integer
- Not closed under $\div$: $(-7) \div 2 = -3.5$
- Commutative: order does not change $+$ or $\times$
- Associative: grouping does not change $+$ or $\times$
- Subtraction is neither: $5 - 3 \neq 3 - 5$

Distributive property and identities

- $a \times (b + c) = a \times b + a \times c$
- $(-2) \times (4 + (-3)) = -8 + 6 = -2$
- 0 is the additive identity: $a + 0 = a$
- 1 is the multiplicative identity: $a \times 1 = a$

 Example

Negative marking in a test

- 1 A test gives +4 per correct answer and –1 per wrong one
- 2 Priya answers 18 correctly and 7 wrongly
- 3 Marks = $18 \times 4 + 7 \times (-1)$
- 4 = $72 + (-7) = 65$ marks

 Quick check

A cold-storage room is at 6°C . It is cooled by 3°C every hour. What is its temperature after 4 hours?

A -12°C

B -6°C

C 6°C

D 18°C

Key takeaways

- ✓ Integers are whole numbers, their negatives and zero
- ✓ On a number line, values increase to the right
- ✓ To subtract, add the opposite
- ✓ $\times$ and $\div$: same signs positive, different signs negative
- ✓ $+$ and $\times$ are commutative and associative