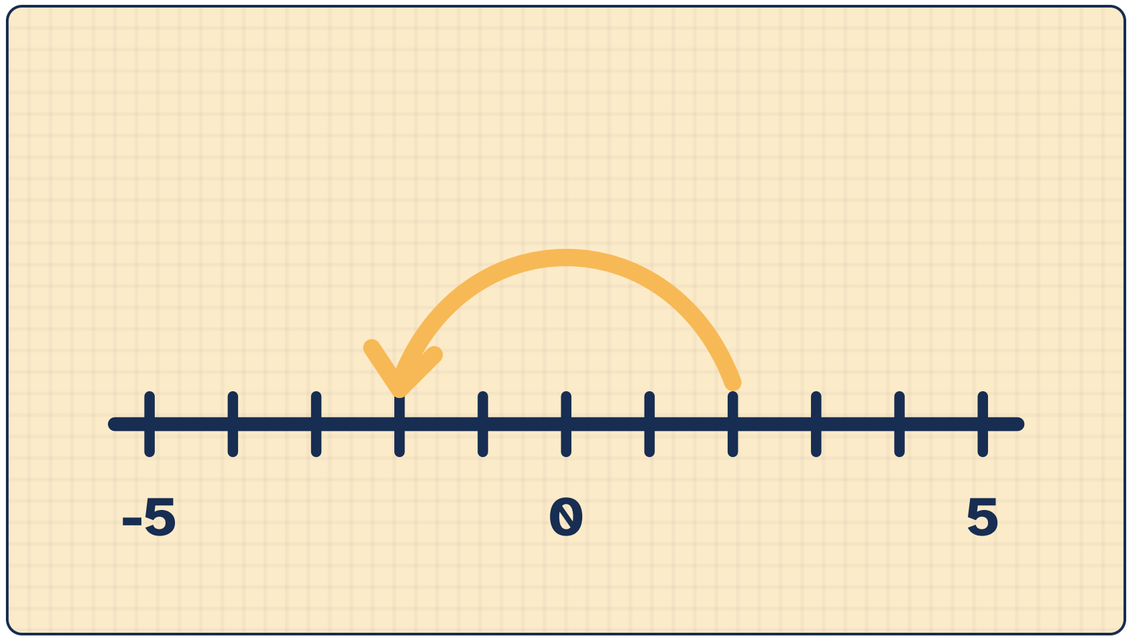
Maths · Number line, the four operations and key properties

Integers



Goals

By the end of this lesson you can

- Show and compare integers on a number line
- Add and subtract integers without sign errors
- Multiply and divide integers using sign rules
- Use the properties of integer operations
- Solve problems on lifts, temperature and money

What are integers?

- Integers: ..., –3, –2, –1, 0, 1, 2, 3, ...
- Positive integers are greater than 0
- Negative integers are less than 0
- Zero is neither positive nor negative
- 2.5 and ½ are not integers

Integers in daily life

- Lift: ground floor is 0, first basement is –1
- Temperature: 5 °C below zero is –5 °C
- Money: a debt of USD 200 is –200
- Height: 50 m below sea level is –50 m

Comparing integers on a number line

- Values increase as you move right
- $-2 > -5$, because -2 lies to the right
- Every negative integer is less than 0
- Every positive integer is greater than every negative

Adding integers: think of a lift

- Add a positive: move up, to the right
- Add a negative: move down, to the left
- Floor 2 + (–3) = –1, one level below ground
- (–4) + (–3) = –7 and 6 + (–8) = –2

Two rules for adding integers

Same signs

- Add the sizes
- Keep the common sign
- $(-4) + (-3) = -7$
- $5 + 2 = 7$

Different signs

- Subtract the smaller size from the larger
- Keep the sign of the larger size
- $(-8) + 5 = -3$
- $9 + (-4) = 5$

Subtracting integers: add the opposite

- The additive inverse of 7 is −7, and of −7 is 7
- a − b means a + (inverse of b)
- 5 − (−3) = 5 + 3 = 8
- (−2) − 4 = (−2) + (−4) = −6
- From 3 °C to −4 °C: (−4) − 3 = −7 °C change

Multiplying integers: spot the pattern

- $3 \times (-4) = -12$ and $2 \times (-4) = -8$
- $1 \times (-4) = -4$ and $0 \times (-4) = 0$
- Each step adds 4, so $(-1) \times (-4) = 4$
- A negative multiplier reverses the direction

Sign rules for × and ÷

Same signs: positive

- (+) × (+) = (+)
- (−) × (−) = (+)
- (−6) × (−5) = 30
- (−100) ÷ (−4) = 25

Different signs: negative

- (+) × (−) = (−)
- (−) × (+) = (−)
- 7 × (−3) = −21
- (−100) ÷ 25 = −4

Dividing integers

- Division undoes multiplication
- $(-100) \div 25$ asks: $25 \times ? = -100$, so -4
- $(-48) \div 6 = -8$ and $(-48) \div (-6) = 8$
- Division by zero is not defined

Early rules for negative numbers

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CE: Brahmagupta wrote rules for multiplying and dividing negatives

- He called positive numbers fortunes and negative numbers debts
- Product of two debts is a fortune: (–) × (–) = (+)

Properties of integer operations

- Closure: +, – and × of integers give an integer
- Not closed under ÷: (–7) ÷ 2 = –3.5
- Commutative: order does not change + or ×
- Associative: grouping does not change + or ×
- Subtraction is neither: 5 – 3 ≠ 3 – 5

Distributive property and identities

- $a \times (b + c) = a \times b + a \times c$
- $(-2) \times (4 + (-3)) = -8 + 6 = -2$
- 0 is the additive identity: $a + 0 = a$
- 1 is the multiplicative identity: $a \times 1 = a$

Worked example

Negative marking in a test

- A test gives +4 per correct answer and –1 per wrong one
- Priya answers 18 correctly and 7 wrongly
- Marks = 18 × 4 + 7 × (–1)
- = 72 + (–7) = 65 marks

Quick check

A cold-storage room is at 6 °C. It is cooled by 3 °C every hour. What is its temperature after 4 hours?

A. –12 °C

B. –6 °C

C. 6 °C

D. 18 °C

Summary

Key takeaways

1. Integers are whole numbers, their negatives and zero
2. On a number line, values increase to the right
3. To subtract, add the opposite
4. × and ÷: same signs positive, different signs negative
5. + and × are commutative and associative