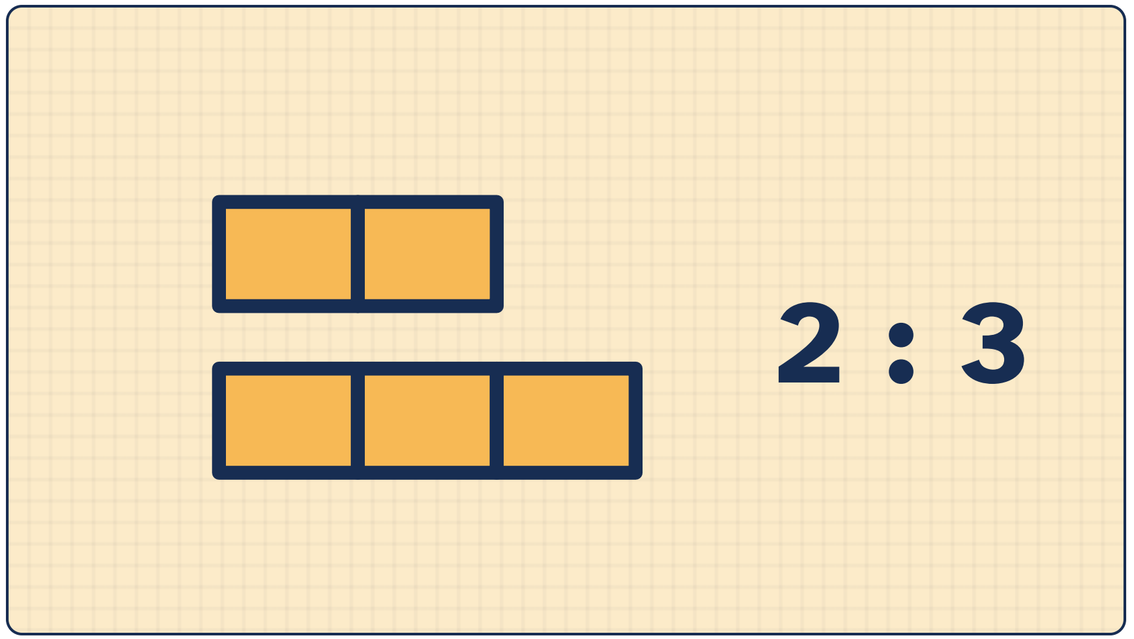
Maths · Simplifying, sharing, unit rates, direct proportion and scale

Ratio and Proportion



Goals

By the end of this lesson you can

- Write and simplify ratios
- Explain how a ratio differs from a fraction
- Share an amount in a given ratio
- Use unit rates and direct proportion
- Use scales on drawings and maps

What is a ratio?

- A ratio compares the sizes of quantities
- 12 red beads and 8 blue beads: red to blue is 12 : 8
- Order matters: blue to red is 8 : 12
- A ratio can have three or more parts, such as 1 : 3 : 5

Worked example

Simplifying ratios

- Divide every part by the highest common factor
- 12 : 8 → divide by 4 → 3 : 2
- Same units first: 40 cm : 2 m = 40 : 200 = 1 : 5
- Clear decimals: 1.5 : 2 → multiply by 2 → 3 : 4

Ratio vs fraction

Ratio: part to part

- Red : blue = 3 : 2
- Compares one part with another
- 3 red beads for every 2 blue

Fraction: part of the whole

- Red = 3/5 of the beads
- Compares one part with the total
- 3 + 2 = 5 parts in the whole

Worked example

Sharing in a ratio

- Share EUR 350 in the ratio 2 : 5
- Total parts: 2 + 5 = 7
- One part: 350 ÷ 7 = EUR 50
- Shares: 2 × 50 = EUR 100 and 5 × 50 = EUR 250
- Check: 100 + 250 = 350

Worked example

Three parts, or one share known

- Share 72 kg in the ratio 1 : 3 : 5
- 9 parts, so one part = 72 ÷ 9 = 8 kg
- Shares: 8 kg, 24 kg and 40 kg
- Boys : girls = 4 : 5 and there are 20 boys
- One part = 20 ÷ 4 = 5, so 25 girls

Unit rates: how much for one?

- A unit rate is an amount per one unit
- 150 km in 2.5 hours: 150 ÷ 2.5 = 60 km/h
- USD 132 for 8 hours: 132 ÷ 8 = USD 16.50 per hour
- Divide by the quantity you want one of

Worked example

Best buy: compare unit prices

- 500 g of rice for GBP 2.40, or 750 g for GBP 3.45?
- Per 100 g: 2.40 ÷ 5 = GBP 0.48
- Per 100 g: 3.45 ÷ 7.5 = GBP 0.46
- The 750 g bag is better value

Direct proportion

- Both quantities change by the same factor
- Double one and the other doubles
- The ratio between them never changes
- Equation: $y = kx$, where k is the unit rate
- Graph: a straight line through $(0, 0)$

Worked example

The unitary method

- 250 g of flour makes 10 biscuits
- How much flour for 18 biscuits?
- One biscuit: 250 ÷ 10 = 25 g
- 18 biscuits: 25 × 18 = 450 g
- Or scale up: 18 ÷ 10 = 1.8, and 250 × 1.8 = 450 g

Proportional or not?

Proportional

- 3 notebooks cost USD 4.50
- 5 notebooks cost USD 7.50
- USD 1.50 each both times

Not proportional

- Taxi: EUR 3 fixed charge plus EUR 2 per km
- 1 km costs EUR 5, 2 km cost EUR 7
- Double the distance, but not double the fare

Worked example

Scale drawings and maps

- Scale 1 : 50 means 1 cm on the plan is 50 cm in real life
- A 3.5 m wall: 350 cm ÷ 50 = 7 cm on the plan
- Map scale 1 : 25,000: 4 cm × 25,000 = 100,000 cm
- 100,000 cm = 1,000 m = 1 km

Common mistakes to avoid

- Writing the parts in the wrong order
- Dividing by one part instead of the total parts
- Mixing units, such as cm with m
- Adding instead of multiplying: 2 : 3 is not 4 : 5

Quick check

Ana and Ben share USD 45 in the ratio 4 : 5. How much does Ben get?

A. USD 25

B. USD 20

C. USD 9

D. USD 36

Summary

Key takeaways

1. Simplify a ratio by dividing by the highest common factor
2. Ratio is part to part; fraction is part to whole
3. Sharing: total parts, one part, then multiply
4. Unit rate = amount for one; compare unit prices
5. Direct proportion: same factor, straight line through (0, 0)